Chap. 26 Design of Digital Controllers

26.1 Digital PID Controller

$$p(t) = \bar{p} + K_c[e(t) + \frac{1}{\tau_I}\int_0^t e(t')dt' + \tau_D \frac{de(t)}{dt}]$$

$$\int_0^t e(t')dt' \cong \sum_{k=1}^{n} e_k \Delta t, \qquad \frac{de}{dt} = \frac{e_n - e_{n-1}}{\Delta t}$$

(rectangular rule) (backward difference)

Position Algorithm

$$p_n = \bar{p} + K_c[e_n + \frac{\Delta t}{\tau_I}\sum_{k=1}^{n} e_k + \frac{\tau_D}{\Delta t}(e_n - e_{n-1})]$$

Let the deviation variable $p_n' = p_n - \bar{p}$

$Z(e_n) = E(z)$, $Z(e_{n-i}) = z^{-i}E(z)$

$$\therefore \quad P'(z) = K_c[E(z) + \frac{\Delta t}{\tau_I}(z^{-n+1} + z^{-n+2} + \ldots + z^{-1} + 1)E(z) + \frac{\tau_D}{\Delta t}(1 - z^{-1})E(z)]$$

$$= K_c[1 + \frac{\Delta t}{\tau_I}(\frac{1}{1 - z^{-1}}) + \frac{\tau_D}{\Delta t}(1 - z^{-1})]E(z) \qquad (\text{as } n \to \infty)$$

$$D(z) = \frac{P'(z)}{E(z)} = K_c[1 + \frac{\Delta t}{\tau_I}(\frac{1}{1 - z^{-1}}) + \frac{\tau_D}{\Delta t}(1 - z^{-1})] \qquad : \text{Position Form}$$

Velocity form (avoid summation, no specification of $\bar{p}$)

$$\Delta p_n = p_n - p_{n-1} = p'_n - p'_{n-1}$$

$$\Delta p_n = K_c[(e_n - e_{n-1}) + \frac{\Delta t}{\tau_I}e_n + \frac{\tau_D}{\Delta t}(e_n - 2e_{n-1} + e_{n-2})]$$

$$\Delta P(z) = K_c[(1 - z^{-1}) + \frac{\Delta t}{\tau_I} + \frac{\tau_D}{\Delta t}(1 - 2z^{-1} + z^{-2})]E(z) \qquad : \text{Velocity Form}$$

- (position form) x $(1 - z^{-1})$ = (velocity form)

For integral, use trapezoidal approximation

$$\int_0^t e(t')dt' \cong \sum_{k=1}^{n}\left(\frac{e_k + e_{k-1}}{2}\right)\Delta t$$

$$\Rightarrow \quad \Delta P(z) = K_c\left[(1 - z^{-1}) + \frac{\Delta t}{2\tau_I}(1 + z^{-1}) + \frac{\tau_D}{\Delta t}(1 - 2z^{-1} + z^{-2})\right]E(z)$$

→ more accurate, but may not achieve better performance
(slightly different values of controller settings)

Features of Digital PID Controller

1. Elimination of Reset windup

- Error summation grows very large ⇒ reset windup

- Saturated controller output ⇒ Sustained error for a while
⇒ big summation term ⇒ errors with opposite sign should cancel the summation over 100% ⇒ cannot act immediately even though the error sign changed!

- Startup situation under automatic control

For Position algorithm,

a. Place an upper limit on the value of summation
(when the controller saturates, suspend the summation until ...)

b. Back calculate the value of e_n that just cause the controller to saturate. (Use this actual value as e_{n-1} in the next controller calc.)

(if saturated,)

$$100 = \bar{p} + K_c\left[e_n + \frac{\Delta t}{\tau_I}\sum_{k=1}^{n-1} e_k + \frac{\Delta t}{\tau_I} e_n + \frac{\tau_D}{\Delta t}(e_n - e_{n-1})\right]$$

$$\Rightarrow \quad e_n = \left[\frac{(100 - \bar{p})}{K_c} - \frac{\Delta t}{\tau_I}\sum_{k=1}^{n-1} e_k + \frac{\tau_D}{\Delta t} e_{n-1}\right] / \left(1 + \frac{\Delta t}{\tau_I} + \frac{\tau_D}{\Delta t}\right)$$

(next)

$$p = \bar{p} + K_c\left[e_n + \frac{\Delta t}{\tau_I}\left(\sum_{k=1}^{n-1} e_k + \frac{(100 - \bar{p})}{k_2} - \frac{\Delta t}{\tau_i}\ . \ . \ .\right) + \frac{\tau_D}{\Delta t}\left(e_n - \frac{\Delta t}{\tau_I}(.\ .\ .)\right)\right]$$

In velocity form : No reset windup problem, but always monitor p_n so that Δp can be ignored if p_n saturates.

2. Elimination of Derivative kick

a. $e_n \rightarrow -b_n$ (if $e_n = r_n - b_n$)

b. Step change in set point $\rightarrow$ ramp
(Limit the rate of change in r_n)

3. Effect of Saturation on Controller Performance

Suppose $\frac{k_c \tau_D}{\Delta t} = 100$ ($\because$ Small Δt) and e_n : 0 - 100 %, p_n : 0 - 100 %

$\Rightarrow$ if Δe_n = 1 %, $\Delta p = 100$ % $\Rightarrow$ exceed the saturation limit
$\Rightarrow$ be careful to select the controller settings and Δt to avoid scaling problem

4. Comparison of Position and velocity Algorithm

Position form	Velocity form
$\bar{p}$ required	X
reset windup	No reset windup
initialization is simple (Just take the signal of final control element)	Pulse counter of stepping motor required
	I-mode is always required (to avoid drift)
	$\Delta p = K_c(e_n - e_{n-1}) = K_c(-b_n + b_{n-1})$ (No setpoint)

5. Use of Dimensionless Controller Gain

K_c will be dimensionless in commercial controller, but input/output values can be shown in engineering unit for convenience $\Rightarrow$ Not dimensionless!!

6. Time delay Compensation

Smith predictor (Model-based approach)
(Digital version : analytical predictor)
$\Rightarrow$ If there are large estimation error (±30%) in time delay, PI will perform better.

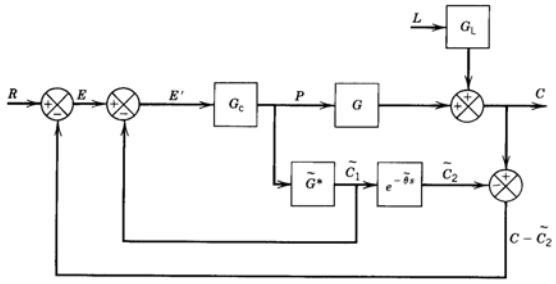


Figure 18.9. Block diagram of the Smith predictor.

$\sim$: model

$\tilde{G}= \tilde{G}^* \cdot e^{-\tilde{\Theta}s}$

$\tilde{G}^*$: w/o time delay

$$E' = E - \tilde{C}_1 = R - \tilde{C}_1 - (C - \tilde{C}_2)$$

if $G=\tilde{G}$, $\tilde{C}_2 = C$ and $E' = R - \tilde{C}_1$ (No time delay in $\tilde{C}_1$)

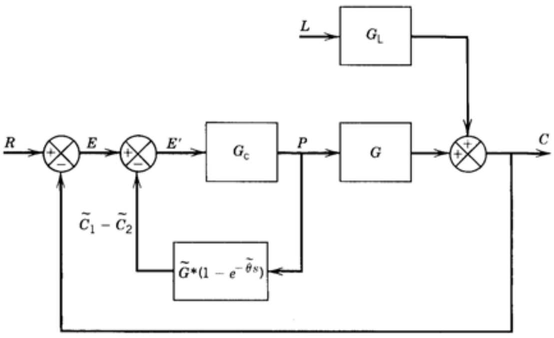


Figure 18.10. An alternative block diagram of a Smith predictor.

$$G_c' = \frac{P}{E} = \frac{G_c}{1+G_c\tilde{G}^*(1-e^{-\tilde{\Theta}s})}$$

For $G=\tilde{G}$,

$$\frac{C}{R} = \frac{G_cG^*e^{-\Theta s}}{1+G_cG^*}$$

by contrast conventional feedback

$$\frac{G_cG^*e^{-\Theta s}}{1+G_cG^*e^{-\Theta s}}$$

for load charge

$$\frac{C}{L} = \frac{G_L[1+G_cG^*(1-e^{-\Theta s})]}{1+G_cG^*}$$

Physical Realizability of Digital Controllers

$$G_c(z) = \frac{b_0+b_1z^{-1}+b_2z^{-2}\ldots.b_kz^{-k}}{a_0+a_1z^{-1}+a_2z^{-2}\ldots.a_mz^{-m}}$$

From above eq'n, $a_0 \neq 0$ → physically realizable
(not depending on future inputs)

26.2 Selection of Digital PID Controller Settings

1. Conversion of Continuous Controller Settings

- for small Δt, finite difference approximation is reasonable
  ($\Delta t/\tau_I \le 0.1$)

- ZOH causes effective one time delay

∴ let $\Theta \to \Theta + \frac{\Delta t}{2}$ and compute K_c, τ_I, τ_D

2. Digital Controllers based on Integral Error Criterion

- ISE, IAE, ITAE

- From the digital simulation, digital PI controller tuning parameters were obtained based on models.

- From the model characteristic (K, Θ, τ)
  ⇒ Find K_c and τ_I depending on Δt

- Lopez *et al* :

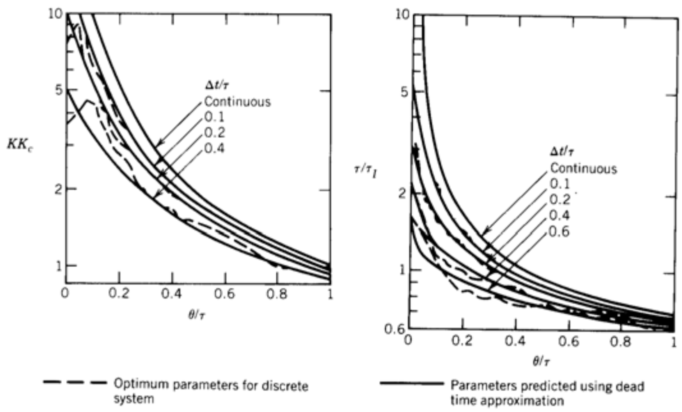


Figure 26.1 Comparison of tuning parameters for a PI controller predicted by the time-delay approximation to those of Lopez [10].

When Θ/τ is very small or $\Delta t/\tau$ is large, K_c must be modified.

- Mosler *et al* : Gain margin = 1.7 and K_C = 0.6 K_{CU}
  For large Δt, P_U and K_C/K_{CU} should be modified.

3. Pole Placement

$$1+G_c(z)HG(z)=0 \Rightarrow (1-a_1z^{-1})(1-a_2z^{-1})\ldots(1-a_nz^{-1})=0$$

Assign desirable a_i's in z-domain

⇒ For continuous system, it's a very good approach, But for discrete system, precaution should be taken since the location of zero is also very important!

⇒ Candidate should be checked by simulation.

26.3 Direct Synthesis Methods

- Closed-loop T. F.

$$\frac{C(z)}{R(z)}=\frac{HG(z)G_c(z)}{1+HG(z)G_c(z)} \quad \text{where} \quad HG(z)=K_mHG_vG_p(z)$$

- If K_m, G_v, G_p, G_c are specified, $\frac{C}{R}$ can be derived from these information.

- In the same manner, specify K_m, G_p, G_v, $(C/R)_d$, then G_c can be calculated.

$$G_c(z)=\frac{1}{HG(z)}\frac{(C/R)_d}{1-(C/R)_d}$$ ($HG(z)$ will be replaced by $\widetilde{HG}(z)$)

- $G_c(z)$ has the reciprocal of process, $\frac{1}{HG(z)}$

- Poles of process become zeros of controller unless some are cancelled by $(C/R)_d$

  ⇒ Stability problem even though the process is stable.

- Theoretically, $G_c(z)\cdot HG(z)=\frac{(C/R)_d}{1-(C/R)_d}$ (No stability problem)

- Actually, $G_c(z)$ cannot cancel the unstable zeros exactly! (due to imperfect models)

- If the system, $HG(z)$ has a time delay, G_c will be have prediction

  ⇒ physical realizability problems

  ⇒ $(C/R)_d$ should contain the z^{-N-1} (-1 is due to effective time delay)

* Minimal Prototype Algorithm

<Design criteria for this algorithm>

1. No offset (requires integral action)
2. Rise time should equal the minimum no. of Δt
3. The settling time should be finite
4. $G_c(z)$ and $(C/R)_d$ must be physically realizable.

- Including a time delay in $(C/R)_d \Rightarrow G_c(z)$ will be physically realizable.

$\therefore \quad (C/R)_d = z^{-N-1} \Rightarrow$ follow the set point exactly except the required time delay

Ex 26.3 (K = 1, a = 0.8187, Δt = 0.2, τ = 1, Θ = 0.2)

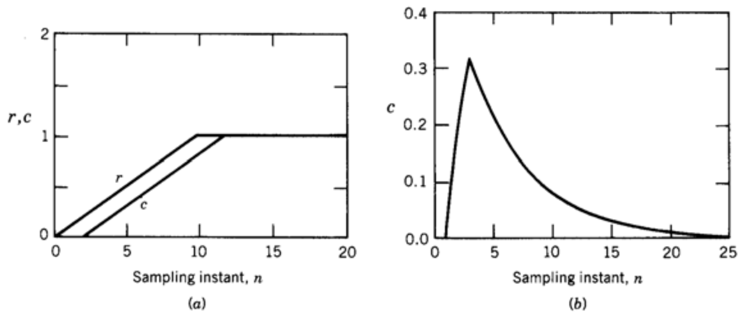
$HG(z) = \dfrac{K(1-a)z^{-N-1}}{1-az^{-2}} \Rightarrow$ develop the minimal prototype controller!

Sol)

let $(C/R)_d = z^{-N-1}$ as ($(C/R)|_{z=1} \rightarrow$ no offset)

$$\therefore \quad G_c(z) = \frac{1-az^{-1}}{K(1-a)z^{-N-1}} \frac{z^{-N-1}}{1-z^{-N-1}} = \frac{(1-az^{-1})}{K(1-a)(1-z^{-N-1})} = \frac{1-0.8187z^{-1}}{0.1813(1-z^{-2})}$$

for ramp change in set point (r_n = $0.1n$, n=0 to 9, r_n = 1 for $n \geq 10$),



for load change

If $L(s) = \dfrac{1}{s}$, $G_L(s) = \dfrac{e^{-\Theta s}}{\tau s+1}$

$$Z[G_L L(s)] = \frac{z^{-N-1}(1-e^{-\Delta t/\tau})}{(1-e^{-\Delta t/\tau}z^{-1})(1-z^{-1})} = \frac{0.1813z^{-2}}{(1-0.8187z^{-1})(1-z^{-1})}$$

$$C(z) = \frac{G_L L(z)}{1+G_c(z)HG(z)} = \frac{G_L L(z)}{1+(C/R)_d/(1-(C/R)_d)}$$

$$= \frac{G_L L(z)(1-(C/R)_d)}{1-(C/R)_d+(C/R)_d} = G_c L(z)(1-(C/R)_d)$$

$$= \frac{0.1813z^{-2}(1-z^{-2})}{(1-0.8187z^{-1})(1-z^{-1})} = \frac{0.1813z^{-2}(1+z^{-1})}{1-0.8187z^{-1}}$$

⇒ $G_c(z)$ has $(1-z^{-N-1})$ in the denominator and for all N, it has also $(1-z^{-1})$ which provides integral action !

Ex 26.4 For $G(s)=1/[(5s+1)(3s+1)]$, (N = 0 and Δt = 1)

$$HG(z) = \frac{(b_1+b_2z^{-1})z^{-N-1}}{1+a_1z^{-1}+a_2z^{-2}}$$ (a_1 = −1.5353. a_2 = 0.5866, b_1 = 0.0280, b_2 = 0.0234)

⇒ minimal prototype controller ?

Sol)

Let $(C/R)_d = z^{-1}$ for $N=0$

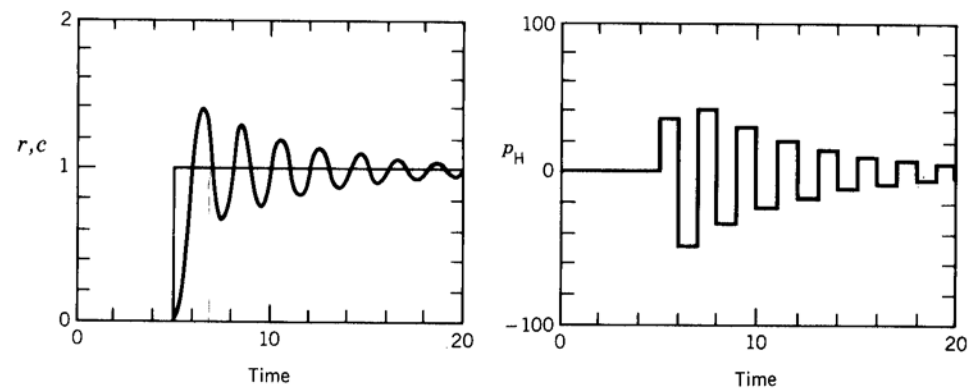
$$G_c(z) = \frac{1}{HG}\frac{(C/R)_d}{1-(C/R)_d} = \frac{1+a_1z^{-1}+a_2z^{-2}}{b_1z^{-1}+b_2z^{-2}}\frac{z^{-1}}{1-z^{-1}}$$

$$= \frac{1+a_1z^{-1}+a_2z^{-2}}{b_1+(b_2-b_1)z^{-1}-b_2z^{-2}} = \frac{1-1.5353z^{-1}+0.5866z^{-2}}{0.0280-0.0046z^{-1}-0.0234z^{-2}}$$

⇒ physically realizable, but intersample ripple (ringing)

$$\frac{(1-0.8195z^{-1})(1-0.7158z^{-1})}{0.028(1-z^{-1})(1+0.8357z^{-1})}$$ (ringing pole in the denominator)

(Not appeared in the output)



<To avoid intersample ripple>

To reach the final steady state, use at least $m+N$ sampling periods

(m : order of denom. $N=\Theta/\Delta t$)

$\Rightarrow$ $(C/R)_d = z^{-N}(\gamma_1 z^{-1} + \gamma_2 z^{-2} + \ldots\ldots + \gamma_m z^{-m})$

where $0 \le \gamma_i \le 1$

Ex) $m=2$, $N=0$ -> $(C/R)_d = \gamma_1 z^{-1} + \gamma_2 z^{-2}$

For step change, $\dfrac{1}{1-z^{-1}}$

$$C = \frac{\gamma_1 z^{-1} + \gamma_2 z^{-2}}{1-z^{-1}} = \gamma_1 z^{-1} + (\gamma_1 + \gamma_2)(z^{-2} + z^{-3} + \ldots\ldots)$$

for no offset -> $\gamma_1 + \gamma_2 = 1$

for no over shoot -> $r_1 < 1$

-> It takes two steps to reach steady state.

For Ex26.4,

$$G_c(z) = \frac{1}{HG}\frac{(C/R)_d}{1-(C/R)_d} = \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{b_1 z^{-1} + b_2 z^{-2}}\frac{\gamma_1 z^{-1} + \gamma_2 z^{-2}}{1 - \gamma_1 z^{-1} - \gamma_2 z^{-2}}$$
$$= \frac{(1 + a_1 z^{-1} + a_2 z^{-2})(\gamma_1 z^{-1} + \gamma_2 z^{-2})}{z^{-1}(b_1 + b_2 z^{-1})(1 - \gamma_1 z^{-1} - \gamma_2 z^{-2})}$$

if $\gamma_2 = 1 - \gamma_1$ ($\gamma_1 < 1$), third term in the denominator is $(1 + (1-\gamma_1)z^{-1})(1 - z^{-1})$

For rippling, examine $\dfrac{P(z)}{R} = \dfrac{G_c(z)E(z)}{R} = \dfrac{1}{HG(z)}\dfrac{C/R \cdot R \cdot (1 - C/R)}{1 - C/R}\dfrac{1}{R}$

$$= \frac{C/R}{HG(z)} = \frac{(1 - a_1 z^{-1} a_2 z^{-2})(\gamma_1 z^{-1} + \gamma_2 z^{-2})}{z^{-1}(b_1 + b_2 z^{-1})}$$ (possible ringing pole in denom.)

$\Rightarrow$ $\gamma_1 = b_1/(b_1 + b_2)$ for no rippling $\Rightarrow$ $(C/R)_d = (b_1 z^{-1} + b_2 z^{-2})/(b_1 + b_2)$

<The disadvantages of the minimal prototype controller>

1. The design specifies only the response at the sampling instants
   --> intersample ripple or large overshoot due to vigorous control action caused by tight specification

2. The controller is highly tuned for the specific type of changes.
   good set point tracking ⇐X⇒ good load regulation

3. force all poles --> around origin of the z-plane for rapid response
   --> extremely sensitive to parameter changes in the model

* Deadbeat Controller (special case of m. p. c with no rippling)

- $C(z)$ will follow $R(z)$ after $(m+N)$ steps exactly (m : tuning parameter)

$$\frac{C}{R} = (p_1 z^{-1} + p_2 z^{-2} + \ldots\ldots + p_m z^{-m}) z^{-N}$$

where $p_1 + p_2 + \ldots\ldots + p_m = 1$

- Others are same as minimal prototype controller
  (finite settling, no offset, physical realizability)

- Controller output, $P(z)$ will stay at the same value after m step for a step change in set point (No rippling)

$$P(z) = (q_0 + q_1 z^{-1} + \ldots\ldots + q_m z^{-m}) R(z) \quad \rightarrow R(z) \text{ needs not be specified}$$

where $q_0 + q_1 + \ldots\ldots + q_m = u(m) = \frac{1}{K_p}$ (1/(steady state gain))

$$G_c = \frac{1}{HG(z)} \cdot \frac{(C/R)_d}{1-(C/R)_d} = \frac{(q_0 + q_1 z^{-1} + \ldots + q_m z^{-m})}{(C/R)_d} \frac{(C/R)_d}{1-(C/R)_d}$$

$$= \frac{q_0 + q_1 z^{-1} + \ldots + q_m z^{-m}}{1 - z^{-N}(p_1 z^{-1} + p_2 z^{-2} + \ldots + p_m z^{-m})} \quad \left(= \frac{P}{C} \cdot \frac{C/R}{1 - C/R} = \frac{P/R}{1 - C/R} \right)$$

$$HG(z) = \frac{C(z)}{P(z)} = \frac{B(z^{-1}) z^{-N}}{A(z^{-1})} = \frac{q_0 (b_1 z^{-1} + b_2 z^{-2} + \ldots + b_m z^{-m}) z^{-N}}{q_0 (1 + a_1 z^{-1} + \ldots + a_m z^{-m})} = \frac{C/R}{P/R}$$

By comparing $HG(z)$ and $1/G_c(z)$,

$q_1 = q_0 a_1$ $\quad p_1 = q_0 b_1$ $\quad \sum_1^m p_i = q_0 (b_1 + b_2 + \ldots\ldots + b_m) = 1$

$q_2 = q_0 a_2$ $\quad p_2 = q_0 b_2$ $\quad q_0 = \frac{1}{b_1 + b_2 + \ldots\ldots b_m} = u(0)$

$q_m = q_0 a_m$ $\quad p_m = q_0 b_m$ $\quad q_i = a_i q_0 = u(i)$

$$\therefore \quad G_c(z) = \frac{q_0 A(z^{-1})}{1 - q_0 B(z^{-1}) z^{-N}}$$

∗ Dahlin's Algorithm (also by Higham independently)

less demanding in terms of closed-loop performance

$$\left(\frac{C}{R}\right)_d = \frac{e^{-hs}}{\lambda s+1}$$ (FOPDT type reference trajectory)

with ZOH, $(C/R)_d = \dfrac{(1-e^{-\Delta t/\lambda})}{1-e^{-\Delta t/\tau} z^{-1}} z^{-N-1}$ (let $\alpha = e^{-\Delta t/\tau}$ and $h = \Theta = N\Delta t$)

$$G_{DC}(z) = \frac{1}{HG(z)} \frac{(C/R)_d}{1-(C/R)_d} = \frac{1}{HG(z)} \frac{(1-\alpha) z^{-N-1}}{1-\alpha z^{-1} - (1-\alpha) z^{-N-1}}$$

The denominator is factored with $(1-z^{-1})$ → integrator → no offset.

λ : tuning parameter

λ↑ : loosening control action (for inaccurate model)

λ→ 0 : G_{DC} = minimal prototype algorithm

Ex. 26.5 For Ex 26.4, set $\lambda = \Delta t = 1$, Find $G_{DC}(z)$

Sol)

$\alpha = e^{-1} = 0.368$ (N=0)

$$G_{DC} = \frac{(1-\alpha) z^{-1}}{1-\alpha z^{-1} - (1-\alpha) z^{-1}} \frac{1}{HG(z)} = \frac{(1-\alpha) z^{-1}}{1-z^{-1}} \frac{1}{HG(z)}$$

$$= \frac{(1-\alpha) z^{-1}}{1-z^{-1}} \cdot \frac{(1+a_1 z^{-1} + a_2 z^{-2})}{(b_1 + b_2 z^{-1}) z^{-1}} = \frac{(1-\alpha)(1+a_1 z^{-1} + a_2 z^{-2})}{(1-z^{-1})(b_1 + b_2 z^{-1})}$$

$$= \frac{0.632(1-1.5353 z^{-1} + 0.5866 z^{-2})}{0.0280(1-z^{-1})(1+0.8357 z^{-1})}$$ (It has ringing pole)

For step change in set point,

$$C(z) = \frac{0.632}{1-0.368 z^{-1}} \cdot \frac{1}{1-z^{-1}}$$

$$\frac{P(z)}{R(z)} = \frac{C/R}{HG(z)} = \frac{0.632(1-1.5353 z^{-1} + 0.5866 z^{-2})}{0.0280(1+0.8357 z^{-1})(1-0.368 z^{-1})}$$

⇒ It shows ringing!

- Modification for ringing in Dahlin's algorithm

⇒ Set z=1 for ringing term

$0.0280(1+0.8357(1)^{-1})=0.0514$

$\Rightarrow \quad \overline{G}_{DC}=\dfrac{0.632(1-1.5353z^{-1}+0.5866z^{-2})}{0.0514(1-z^{-1})}$

$\Rightarrow$ Then, it's hard to predict the closed-loop behavior

- disadvantage : lack of predictability instead of ringing

* An analysis of Ringing

Ringing cause excessive wear in actuator
(unique to discrete time direct synthesis method)

Let $G_c(z)=\dfrac{1}{1-p_1z^{-1}}G_c'(z)$ (where $G_c'(z)$ is the T.F. excluding the factor)

then

$$P(z) =[\frac{1}{1-p_1z^{-1}}G_c'(z)]E(z)$$
$$=\frac{r_1}{1-p_1z^{-1}}+\text{Other Terms}$$

$\Rightarrow \; P(n\Delta t)=r_1\,(p_1)^n+$ Other Terms

if $p_1<0$, $r_1\,(p_1)^n$ alternates its sign as n increases

if p_1 is near origin, the ringing may not be noticeable.

The nonringing version of $\overline{G}_{DC}$ for Ex 26.4

(since $(C/R)_d\neq C/R$ for nonringing version)

$$\frac{P(z)}{R(z)}=\frac{\overline{G}_{DC}(z)E(z)}{R(z)}=\overline{G}_{DC}(z)(1-\frac{C(z)}{R(z)})=\overline{G}_{DC}(1-\frac{HG(z)\cdot\overline{G}_{DC}(z)}{1+HG(z)\,\overline{G}_{DC}(z)})$$

$$=\frac{\overline{G}_{DC}(z)}{1+HG(z)\,\overline{G}_{DC}(z)}$$

$$=\frac{\dfrac{(1-\alpha)(1+a_1z^{-1}+a_2z^{-2})z^{-N-1}}{(b_1+b_2)(1-\alpha z^{-1}-(1-\alpha)z^{-N-1})}}{1+\dfrac{(b_1+b_2z^{-1})z^{-N-1}}{(1+a_1z^{-1}+a_2z^{-2})}\cdot\dfrac{(1-\alpha)(1+a_1z^{-1}+a_2z^{-2})z^{-N-1}}{(b_1+b_2)(1-\alpha z^{-1}-(1-\alpha)z^{-N-1})}}$$

$$=\frac{(1-\alpha)(1+a_1z^{-1}+a_2z^{-2})}{(b_1+b_2)(1-\alpha z^{-1}-(1-\alpha)z^{-N-1})+(b_1+b_2z^{-1})(1-\alpha)z^{-N-1}}$$

If $N=0$, the denominator becomes

$$(b_1 + b_2)(1-\alpha z^{-1} - z^{-1} + \alpha z^{-1}) + (b_1 + b_2 z^{-1})(1-\alpha) z^{-1}$$
$$= (b_1+b_2) - (\alpha b_1 + b_2) z^{-1} + b_2(1-\alpha) z^{-2}$$

and ringing pole may exist.

If $N=1$,an additional ringing pole could apprear

As N increases, several ringing poles can apprear additionally.

* Vogel-Edgar Algorithm

Eliminate the ringing pole due to $HG(z)$ for process

$$\left(\frac{C}{R}\right)_d = \frac{e^{-hs}}{\lambda s+1} \cdot \frac{N(HG(z))}{N(HG(1))}$$ (denominator for offset-free response)

For $\dfrac{(b_1+b_2 z^{-1}) z^{-N-1}}{1+a_1 z^{-1} + a_2 z^{-2}}$, $\alpha = e^{-\Delta t/\lambda}$

$$\left(\frac{C}{R}\right)_d = \frac{(1-\alpha) z^{-N-1}}{1-\alpha z^{-1}} \cdot \frac{b_1+b_2 z^{-1}}{b_1+b_2}$$ -> may slow down

$$G_{VE}(z) = \frac{1}{HG(z)} \frac{(C/R)_d}{1-(C/R)_d} = \frac{1+a_1 z^{-1} + a_2 z^{-2}}{(b_1+b_2 z^{-1}) z^{-N-1}} \frac{\dfrac{(1-\alpha) z^{-N-1}}{1-\alpha z^{-1}} \dfrac{b_1+b_2 z^{-1}}{b_1+b_2}}{1-\dfrac{(1-\alpha) z^{-N-1}}{(1-\alpha z^{-1})} \dfrac{b_1+b_2 z^{-1}}{b_1+b_2}}$$

$$= \frac{(1+a_1 z^{-1} + a_2 z^{-2})(1-\alpha)}{(b_1+b_2)(1-\alpha z^{-1}) - (1-\alpha)(b_1+b_2 z^{-1}) z^{-N-1}}$$

If $a_2 = b_2 = 0$ (a 1st order process), $\left(\dfrac{C}{R}\right)_d = \dfrac{(1-\alpha) z^{-N-1}}{1-\alpha z^{-1}}$

-> Dahlin's algorithm

$$\frac{P(z)}{R(z)} = \frac{(C/R)_d}{HG(z)} = \frac{(1-\alpha)(1+a_1 z^{-1} + a_2 z^{-2})}{(b_1+b_2)(1-\alpha z^{-1})}$$

-> No ringing problem ! (since α>0)

also λ is a tuning parameter!

If ringing poles appear, $G_{DC} < \overline{G}_{DC} < G_{VE}$ (better)

If no ringing poles appear, $\overline{G}_{DC} < G_{DC} < G_{VE}$ (better)

G_{VE} can handle systems with positive zero as well as with simulated noise.

G_{VE} and G_{DC} cannot handle unstable process due to imperfect process model..

G_{VE} shows good robustness (than G_{DC})

* Internal Model Control (IMC)

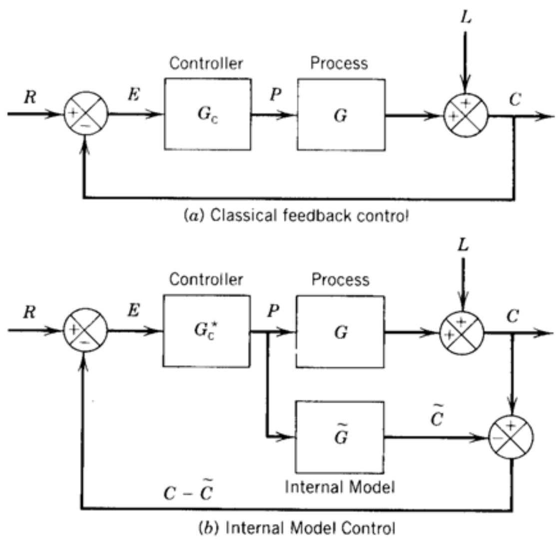


Figure 12.2. Feedback control strategies.

The controller :

$$G_c^* = \frac{1}{\tilde{G}_-} \cdot f$$

$\tilde{G} = \tilde{G}_+ \cdot \tilde{G}_-$ (Stable process)

where

$\tilde{G}_+$: contains time delays and "OUC" zero with unit gain + "IUC" zero near (-1,0)

$\tilde{G}_-$: "IUC" zero and poles (invertible part)

f : low pass filter $= \dfrac{1}{(\tau_c s + 1)^r}$
for physical realizability and robustness
-> Tuning parameters

$$C = \frac{G_c^* G}{1 + G_c^*(G - \tilde{G})} R + \frac{1 - G_c^* \tilde{G}}{1 + G_c^*(G - \tilde{G})} L$$

If $G = \tilde{G}$, $\quad C = G_c^* G R + (1 - G_c^* G) L$

$\Rightarrow \; C/R = \tilde{G}_+(z) f(z)$

Ex 26.6 Design an IMC for $\tilde{G}(s) = e^{-2s}/(5s+1)$

For $\Delta t = 1$, $H\tilde{G}(z) = \dfrac{0.1813 z^{-3}}{1 - 0.8187 z^{-1}}$

sol) $\tilde{G}_+(z) = z^{-3}$, $\quad \tilde{G}_-(z) = \dfrac{0.1813}{1 - 0.8187 z^{-1}}$

$f(z) = \dfrac{1-\alpha}{1 - \alpha z^{-1}}$ (α is a tuning parameter)

$G_c^*(z) = \dfrac{1 - 0.8187 z^{-1}}{0.1813} \dfrac{(1-\alpha)}{1 - \alpha z^{-1}}$: IMC (lead - lag structure)

$\Rightarrow \; (C/R) = z^{-3} \dfrac{(1-\alpha)}{1 - \alpha z^{-1}}$

if $\alpha = e^{-\Delta t/\lambda}$ → Dahlin's controller

if $\alpha = 0$ → Deadbeat IMC → minimal prototype controller.

26.4 Digital Feedforward Control

- Use with feedback controller normally

- The measured value of disturbance should be available

- Select G_f so that $C=R=0$ (disturbance is cancelled)
  $\Rightarrow$ $C(n\Delta t)=0$ at the sampling instants
  (No specification on the response of intersampling period)

- Perfect control may not be attainable due to physical realizability and imperfect model even with continuous version

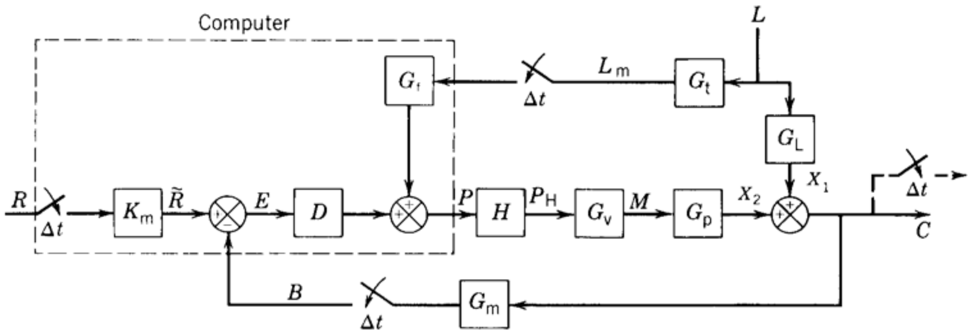


Figure 26.9 Block diagram for digital feedback/feedforward control.

$\Rightarrow$ $LG_L(z)+LG_t(z)G_f(z)HG_vG_p(z)=0$

$$\therefore\quad G_f=\frac{-LG_L(z)}{LG_t(z)HG_vG_p(z)}$$

- Since $LG_L(z)/LG_t(z)\neq G_L(z)/G_t(z)$, in general, G_f cannot be obtained unless L is specified. (For the continuous case, L will be cancelled)

- However, $HG_L/HG_t=LG_L(z)/LG_t(z)$ if L is piecewise constant
  $\Rightarrow$ Assumed that L can be approximated by a piecewise constant function.

- If $G_t(s)=K_te^{-\Theta_ts}$ and $\Theta=T\Delta t$, $HG_t(z)=K_tz^{-T}$

$$\Rightarrow\quad G_f(z)=\frac{-HG_L(z)}{K_tz^{-T}HG_vG_p(z)}$$

- For realizability $\Theta_L>(\Theta_t+\Theta_p)$ (where Θ_p is the process time delay)
  $\Rightarrow$ Before L affects output through load T. F., MV must act on the process output!

<Other approach>

- Use a (lead-lag + time-delay) type controller

$$G_f(z)=\frac{K_f(1+b_f z^{-1})}{1+a_f z^{-1}} z^{-N_f}$$

Tuning parameters : K_f, a_f, b_f, N_f

- find $G_f(s)$ in continuous time domain, and then discretize!

Ex.26.7 A small distillation column separating methanol and water is controlled by reflux flow rate for the methanol composition on the top. The major disturbance is the composition of the feed.

$$G_v G_p=\frac{-5e^{-4s}}{(5s+1)(3s+1)} \quad , \; G_t=0.2$$

$$G_L=\frac{1.5e^{-4s}}{(7s+1)(2s+1)} \quad \text{(time unit : minute)}$$

Δt = 1min, L is changed in a piecewise constant manner.

a) Dynamic feedforward Controller

$$G_f=\frac{-HG_L(z)}{HG_t HG_v G_p(z)},$$

$$HG_v G_p(z)=\frac{(-0.1399z^{-1}-0.1171z^{-2})z^{-4}}{1-1.5353z^{-1}+0.5866z^{-2}}$$

$$HG_L(z)=\frac{(0.0435z^{-1}+0.0351z^{-2})z^{-4}}{1-1.4734z^{-1}+0.5258z^{-2}} \qquad G_t=K_t=0.2$$

$$G_f(z)=\frac{-0.2174z^{-1}+0.1583z^{-2}+0.1419z^{-3}-0.1029z^{-4}}{-0.1399z^{-1}+0.089z^{-2}+0.099z^{-3}-0.0616z^{-4}}$$

zeros ; $-0.2174(1-0.8208z^{-1})(1+0.8071z^{-1})(1-0.7144z^{-1})$

poles ; $-0.1399(1-0.8663z^{-1})(1+0.8372z^{-1})(1-0.6071z^{-1})$

⇒ small size ringing can appear! (Not much noticeable)

b) Steady state feedforward controller : $G_f(z\to 1)=1.5$

c) lead-lag type FF controller ($N_f = 0$)

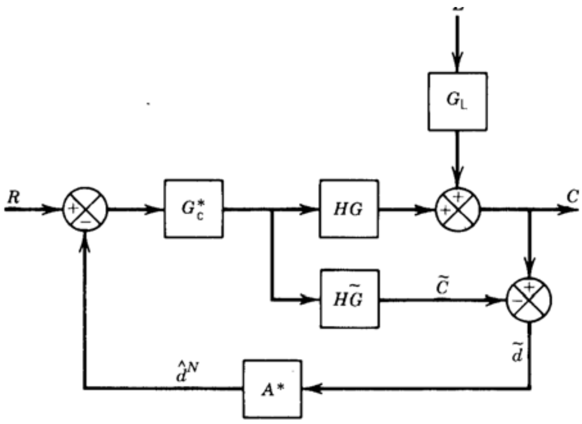
$$\frac{K_f(1+b_f)}{(1+a_f)} = 1.5 \Rightarrow \text{Choose } a_f \text{ and } b_f$$

⇒ by visual tuning a_f = -0.9, b_f = -0.89

$$G_f = \frac{1.0112(1-0.89z^{-1})}{1-0.9z^{-1}}$$

26.5 Combined Load Estimation and Time-delay Compensation

* **Analytical Predictor (Doss and Moore, 1982)**



HG = actual process
$\widetilde{HG}$ = process model
A^* = disturbance predictor/filter
G_c^* = controller
G_L = load transfer function
$\hat{d}^N$ = $\tilde{d}$ predicted N steps ahead

Figure 26.12 Model-predictive block diagram for analytical predictor

Suppose L is a step function (HG_L can be used) and $HG_L(z) = HG(z) = HG^*(z)z^{-N}$ and perfect model is used, Then

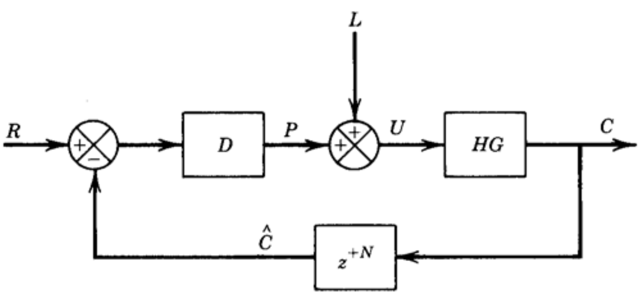


Figure 26.13 Alternative block diagram for the analytical predictor.

$$C(z) = \frac{G_c(z)HG(z)}{1+G_c(z)HG^*(z)}R(z) + \frac{HG(z)}{1+G_c(z)HG^*(z)}L(z)$$

⇒ Char. equation does not contain a time delay

⇒ increased stability margin of $G_c(z)$

⇒ z^{+N} should be implemented indirectly!

⇒ AP can handle only if $HG_L = HG_P$ -> if not, use GAP!

*** Generalized Analytical Predictor (GAP)**

Assume $\tilde{G}$ and G_L are FOPDT and No modeling error ($G=\tilde{G}$)

⇒ $\tilde{d}(z) = C(z) - \tilde{C}(z)$

⇒ $\hat{d}^N(z)$ should be estimated

- Assume L is constant over the prediction horizon N (or $N+1$) and

$H\tilde{G}_L(z) = \dfrac{\bar{b}z^{-1}}{1-\bar{a}z^{-1}}$ (No time delay, 1st order) ($\tilde{G}_L \neq \tilde{G}$ not as in AP)

$\tilde{d}(z) = HG_L(z)L(z)$ if $G=\tilde{G}$ ($H\tilde{G}_L \to HG_L$)

⇒ $\tilde{d}(z) = \dfrac{\bar{b}z^{-1}}{1-\bar{a}z^{-1}}L(z)$ ⇒ $(1-\bar{a}z^{-1})\tilde{d}(z) = \bar{b}z^{-1}L(z)$ ⇒ $\tilde{d}_k = \bar{a}\,\tilde{d}_{k-1} + \bar{b}L_{k-1}$

- Assume a step change in load occurred at time $k-1$

⇒ estimated $\hat{L}_{k-1} = \dfrac{1}{\bar{b}}(\tilde{d}_k - \bar{a}\,\tilde{d}_{k-1})$ ⇒ $z^{-1}\hat{L}(z) = \dfrac{1-\bar{a}z^{-1}}{\bar{b}}\tilde{d}(z)$

$\hat{d}_{k+1} = \bar{a}\,\tilde{d}_k + \bar{b}\,\hat{L}_{k-1}$

$\hat{d}_{k+N} = \bar{a}\,\hat{d}_{k+N-1} + \bar{b}\,\hat{L}_{k-1}$

⇒ $\hat{d}_{k+N} = \bar{a}^N\,\tilde{d}_k + \dfrac{(1-\bar{a}^N)}{(1-\bar{a})}\bar{b}\,\hat{L}_{k-1}$ ($\tilde{d}_k$: measured; $\hat{L}_{k-1}$: constant)

- Since $\tilde{d}_k$ data may be noisy, use filter for $\hat{L}(z)$

$z^{-1}\hat{L}(z) = F_L(z)\dfrac{(1-\bar{a}z^{-1})}{\bar{b}}\tilde{d}(z)$

where $F_L(z) = \dfrac{1-\beta}{1-\beta z^{-1}}$, $0 \le \beta < 1$

first order filter tuning parameter

- From Block Diagram,

$$\hat{d}^N(z) = A^*(z)[C(z) - \tilde{C}(z)]$$

$$= \bar{a}^N \tilde{d}(z) + \frac{1-\bar{a}^N}{1-\bar{a}} \bar{b}(z^{-1}\hat{L}(z))$$

$$= \bar{a}^N [C(z) - \tilde{C}(z)] + \frac{1-\bar{a}^N}{1-\bar{a}} F_L(z)(1-\bar{a}z^{-1})[C(z) - \tilde{C}(z)]$$

$$= \left\{ \bar{a}^N + \frac{1-\bar{a}^N}{1-\bar{a}} F_L(z)(1-\bar{a}z^{-1}) \right\} [C(z) - \tilde{C}(z)]$$

$$\therefore \ A^*(z) = \bar{a}^N + \frac{1-\bar{a}^N}{1-\bar{a}} F_L(z)(1-\bar{a}z^{-1})$$

- Also, $G_C{}^*$ can be designed by IMC design procedure.

Ex. 26.8 $G(s) = \dfrac{e^{-2s}}{5s+1}$ is controlled using an IMC controller ($\Delta t = 1$)

Assume $G = \tilde{G}$, $G_L = \dfrac{1}{s+1}$, $L = \dfrac{1}{s} e^{-5s}$

sol) IMC : $G_c{}^* = \dfrac{1-0.8183z^{-1}}{0.1817}$ for $\alpha = 0$ (deadbeat design)

($A^* = 1$, $\tau_L = 0$)

$\overline{HG_L}(z) = \dfrac{(1-a_L)z^{-1}}{1-a_L z^{-1}}$ ($a_L = e^{-\Delta t/\tau_L}$)

AP : using $H\tilde{G}_L = H\tilde{G}$ (model error $\tau_L = 5$) : overcorrection

GAP : perfect prediction and $\tau_L = 1$: No modeling error
(better due to load prediction)
if G_L is FOPDT, only τ_L needs to be known
($\because$ $A^* = f(\bar{a})$ even though GAP need load filtering)